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Tracking Problem for Induction Electric Drive under Influence of Unknown Perturbation ⋆

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Abstract: The tracking problem for induction motor drives is considered in the paper. The main attention is devoted to the problem of unknown load torque compensation not belonging to the control space. The model of induction motor is considered in the fixed reference frame concerned with the stator. A modification of vector control principle is designed on the base of new relay control law, which is used for generation of damping oscillating modes with unlimited growth in oscillation frequency. The theoretically infinite coefficient of relay linearization enabling asymptotic invariance of the output to a wide class of external perturbations. The simulation results show the efficiency of proposed approach.

Keywords: stability of nonlinear systems, discontinuous control, induction motor.

1. INTRODUCTION

Most of the used electric motors are induction with squirrel cage rotor Krause (1986). Their widespread use is primarily conditioned by ease of maintenance and operation, simplicity of design, low cost and high reliability. List of their applications can be very long: electric drives of smoke exhausters, cranes, ball mills, pumps, conveyors, winches and so on. On the other hand induction motor is the most complicated electric drive from the point of view of control theory Leonhard (1990) due to the following subjects: its dynamics is strongly nonlinear; parameters of the motor (rotor and stator resistance) and load torque can vary significantly.

There are wide range of control strategies for electric motor drives and in many papers designers draw attention to unknown load torque attenuation and parameter uncertainties compensation. In Marino et al. (2008); Harnefors et al. (2010) an adaptive technique is used to compensate unknown constant parameters and load torque variation. Such technique can be combined with decomposition approaches proposed in Traore et al. (2012); Shieh and Shyu (1999) on the basis of backstepping procedure or passivity control Gokdere and Simaan (1997). Another group of the papers are concerned with sliding mode control of induction motor. In Shieh and Shyu (1999); Yan et al. (2000) a closed-loop speed and flux estimator based on the sliding-mode control methodology is proposed. Applying the estimator, the corresponding sliding-mode torque controller is designed and it is shown that speed control can be achieved by using the torque controller.

Instead of using the methods discussed in the above papers, the aim of this paper is to solve the problem of unknown arbitrary load torque compensation on the basis of static feedback. The basic algorithm is based on the new results in relay control theory Kochetkov and Utkin (2013). Theory of relay system is well known for control engineers Tsypkin (1984). In particular, the most part of modern electric drives are supplied by power electronic converters that have switching nature, and using to stable on/off state. This fact is concerned with simplicity of practical realization of bang-bang control Dodds et al. (1998). Also, the relay system are often used as basis for sliding mode control algorithm Utkin et al. (2009). It is well known, that motion on sliding surface does not depend on matched disturbances Drazenovic (1969). The motivation for producing the system presented here originated with the problem of independently controlling speed and rotor flux of induction motor drive under influence of unmatched disturbances. This task is studied widely in different problem statement Utkin (2001). The main attention of designers is devoted to the mentioned problem under influence of external and parametric perturbations, which physically are the unknown load deviation, motor parameters changing with time and etc. The paper is organized as follows. In section 2 the model of induction motor is introduced, the class of external load torque and reference signals for rotor speed and magnetic flux are discussed and finally problem statement is stated. The proposed relay controller is synthesized in section 3. The simulation results of section 4 show the efficiency of proposed algorithm. Finally, some concluding remarks end the paper.

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Parameter/ Variable	Definition
L_s	stator inductance
L_r	rotor inductance
L_m	rotor/stator mutual inductance
R_s	stator resistance
R_r	rotor resistance
T_r	rotor time constant, L_r/R_r
p	number of stator pole pairs
J	rotor moment of inertia
$I_{s\alpha}$	α component of stator current
$I_{s\beta}$	β component of stator current
$\psi_{r\alpha}$	α component of stator flux linkage with rotor
$\psi_{r\beta}$	β component of stator flux linkage with rotor
$u_{s\alpha}$	α axis component of stator voltage
$u_{s\beta}$	β axis component of stator voltage
Γ	torque developed by the motor
Γ_L	load torque of driven mechanical load
ω	mechanical rotor speed

Table 1. Variables and constant parameters of induction motor and mechanical load

2. PROBLEM STATEMENT

Let us consider the following standard set of induction motor equations formulated in the α_β co-ordinated system Krause (1986):

$$\begin{aligned}
\dot{\omega} &= \frac{1}{J}[\Gamma - \Gamma_L(t)] = \\
&= \frac{1}{J}\left\{\frac{3L_m}{2L_r}p(\psi_{r\alpha}I_{s\beta} - \psi_{r\beta}I_{s\alpha}) - \Gamma_L(t)\right\} \\
\dot{\psi}_{r\alpha} &= -\frac{1}{T_r}\psi_{r\alpha} - p\omega\psi_{r\beta} + \frac{L_m}{T_r}I_{s\alpha}, \\
\dot{\psi}_{r\beta} &= -\frac{1}{T_r}\psi_{r\beta} + p\omega\psi_{r\alpha} + \frac{L_m}{T_r}I_{s\beta}, \\
\dot{I}_{s\alpha} &= \frac{L_r}{L_sL_r - L_m^2}\left\{u_{s\alpha} - \left(R_s + \frac{L_m^2}{L_rT_r}\right)I_{s\alpha} + \right. \\
&\left. + \frac{L_m}{L_r}\left[\frac{1}{T_r}\psi_{r\alpha} + p\omega\psi_{r\beta}\right]\right\}, \\
\dot{I}_{s\beta} &= \frac{L_r}{L_sL_r - L_m^2}\left\{u_{s\beta} - \left(R_s + \frac{L_m^2}{L_rT_r}\right)I_{s\beta} + \right. \\
&\left. \frac{L_m}{L_r}\left[\frac{1}{T_r}\psi_{r\beta} - p\omega\psi_{r\alpha}\right]\right\},
\end{aligned} \tag{1}$$

where the variables and constant parameters are defined in table 1.

It is assumed that load torque of unknown mechanical load Γ_L is arbitrary bounded function with two bounded derivatives

$$\left|\Gamma_L^{(i)}(t)\right| \le \Gamma_b^i,\ i = \overline{0,2}, \tag{2}$$

where $\Gamma_L^{(i)}(t)$ denotes the i-th derivative of $\Gamma_L(t)$, $\Gamma_b^i =$ const > 0 are known constants, $|\cdot|$ hereafter denotes the absolute value of a number.

Physically the load torque Γ_L comprises an external load and a dynamic load torque, representing the dynamics of any driven mechanism.

Let us rewrite system (1) in more compact form

$$\begin{aligned}
\dot{\omega} &= \frac{1}{J}[c_5\psi^{\mathrm{T}}T^{\mathrm{T}}I - \Gamma_L(t)], \\
\dot{\psi} &= -P(\omega)\psi + c_4I, \\
\dot{I} &= c_1[c_2P(\omega)\psi - a_1I + U],
\end{aligned} \tag{3}$$

where $\psi^{\mathrm{T}} = [\psi_{r\alpha}\ \psi_{r\beta}]$, $I^{\mathrm{T}} = [I_{s\alpha}\ I_{s\beta}]$, $c_1 = \dfrac{L_r}{L_sL_r - L_m^2}$, $U^{\mathrm{T}} = [u_{s\alpha}\ u_{s\beta}], c_2 = \dfrac{L_m}{L_r}, c_4 = \dfrac{L_m}{T_r}, c_5 = \dfrac{3pL_m}{2L_r}$, $a_1 = R_s + \dfrac{L_m^2}{L_rT_r}$.

Also,

$$P(\omega) = \begin{pmatrix} c_3 & p\omega \\ -p\omega & c_3 \end{pmatrix}, T = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}, \tag{4}$$

where $c_3 = 1/T_r$.

Let us introduce into consideration some desired values of the rotor speed and the rotor magnetic flux

$$\begin{aligned}
&\omega_d(t) = \text{var},\ \psi_d(t) = \text{var} > 0, \\
&|\omega_d^{(i)}(t)| \le W_d^i,\ |\psi_d^{(i)}(t)| \le \Psi_d^i,\ i = \overline{0,2}
\end{aligned} \tag{5}$$

where $W_d^i =$ const > 0, $\Psi_d^i =$ const > 0 are known constants, $\omega_d^{(i)}(t)$, $\psi_d^{(i)}(t)$ denote the i-th derivatives of the variables $\omega_d(t)$, $\psi_d(t)$, which numerical values are assumed to be known exactly.

Under assumption, that all state space variables are measured in system (3) the tracking problem is stated in the paper

$$\lim_{t\to\infty} |\overline{\omega}| = 0,\ \lim_{t\to\infty} |\overline{\psi}| = 0, \tag{6}$$

where $\overline{\psi} = ||\psi|| - \psi_d(t)$, $||\psi|| = \psi_{r\alpha}^2 + \psi_{r\beta}^2$, $\overline{\omega} = \omega - \omega_d(t)$.

The control law will be developed in detail in the next sections.

3. CONTROL ALGORITHM SYNTHESIS

3.1 Main Result

The control law will be developed in detail in the next sections. The control law synthesis procedure is based on step by step decomposition technique. Firstly, we choose a fictitious control in the mechanical subsystem, than the real control is chosen in discontinuous form on the base of so called vortex algorithm Kochetkov and Utkin (2013). To provide independent control of the rotor magnetic flux and rotor speed, a decomposition procedure is used. On the first step of our decomposition procedure we introduce the new variables and proof convergence of the inner loop controller under some conditions. To provide the relations from first step, we choose real control input on the second step on the base of sliding mode technique.

Step 1. To provide solution of our problem, let us write equation for the rotor magnetic flux with the help of (3)–(6)

$$\frac{d||\psi||}{dt} = \psi^{\mathrm{T}}\dot{\psi} + \dot{\psi}^{\mathrm{T}}\psi = -2(c_3||\psi|| - c_4\psi^{\mathrm{T}}I).$$

This together with first equation of the system (3), constitute the motor differential equations to be used to synthesize the fictitious control inputs (the stator current components)

$$\begin{aligned}\dot{\overline{\omega}} &= \frac{1}{J}[c_5\psi^{\mathrm{T}}T^{\mathrm{T}}I - \Gamma_L(t)] - \omega_d^{(1)}(t),\\ \dot{\overline{\psi}} &= -2(c_3||\psi|| - c_4\psi^{\mathrm{T}}I) - \psi_d^{(1)}(t).\end{aligned} \tag{7}$$

Firstly, we synthesize inner control loop under unknown load torque (2). Let us introduce the coordinate transformation

$$\begin{aligned}&\frac{1}{J}c_5\psi^{\mathrm{T}}T^{\mathrm{T}}I = \widetilde{I} + \omega_d^{(1)}(t),\\ &-2(c_3||\psi|| - c_4\psi^{\mathrm{T}}I) = \widetilde{\psi} + \psi_d^{(1)}(t)\end{aligned} \tag{8}$$

with variables $\widetilde{I}$, $\widetilde{\psi}$ are governed by the following equations

$$\begin{aligned}\dot{\widetilde{I}} &= -\alpha_1\widetilde{I} - \beta_1\overline{\omega} - M_1\mathrm{sign}(\overline{\omega}),\\ \dot{\widetilde{\psi}} &= -\alpha_2\widetilde{\psi} - \beta_2\overline{\psi} - M_2\mathrm{sign}(\overline{\psi}),\end{aligned} \tag{9}$$

where $\alpha_i = \mathrm{const} > 0$, $\beta_i = \mathrm{const} > 0$, $M_i = \mathrm{const} > 0$ are inner loop controller parameters, $\mathrm{sign}(\cdot)$ is the sign function, which is determined in the Fillipov's sense, for example,

$$\mathrm{sign}(\overline{\omega}) = \begin{cases} 1, \ if \ \ \overline{\omega} > 0;\\ -1, \ if \ \ \overline{\omega} < 0;\\ \in [-1,\, 1], \ if \ \ \overline{\omega} = 0.\end{cases}$$

The inner controller is realized in the computational environment. Due to this reason, one can choose arbitrary initial conditions for the integrators in (9). For further real control inputs synthesis, let us choose in the following way

$$\begin{aligned}\widetilde{I}(t_0) &= \frac{1}{J}c_5\psi^{\mathrm{T}}(t_0)T^{\mathrm{T}}I(t_0) - \omega_d^{(1)}(t_0),\\ \widetilde{\psi}(t_0) &= -2[c_3||\psi(t_0)|| - c_4\psi^{\mathrm{T}}(t_0)I(t_0)] -\\ &\quad -\psi_d^{(1)}(t_0),\end{aligned} \tag{10}$$

where t_0 is initial moment of time.

From (7)–(9) the following equations with respect to tracking errors can be derived

$$\begin{aligned}\dot{\overline{\omega}} &= \widetilde{I} - \frac{\Gamma_L(t)}{J},\\ \dot{\widetilde{I}} &= -\alpha_1\widetilde{I} - \beta_1\overline{\omega} - M_1\mathrm{sign}(\overline{\omega}),\\ \dot{\overline{\psi}} &= \widetilde{\psi},\\ \dot{\widetilde{\psi}} &= -\alpha_2\widetilde{\psi} - \beta_2\overline{\psi} - M_2\mathrm{sign}(\overline{\psi}).\end{aligned}$$

Finally, let us introduce the variable $\widetilde{I}^* = \widetilde{I} - \Gamma_L(t)/J$ and rewrite the last system in the form

$$\begin{aligned}\dot{\overline{\omega}} &= \widetilde{I}^*,\\ \dot{\widetilde{I}}^* &= -\alpha_1\widetilde{I}^* - \beta_1\overline{\omega} - M_1\mathrm{sign}(\overline{\omega}) + \xi(t),\\ \dot{\overline{\psi}} &= \widetilde{\psi},\\ \dot{\widetilde{\psi}} &= -\alpha_2\widetilde{\psi} - \beta_2\overline{\psi} - M_2\mathrm{sign}(\overline{\psi}),\end{aligned} \tag{11}$$

where $\xi(t) = -\alpha_1\Gamma_L(t)/J - \Gamma_L^{(1)}(t)/J$.

Taking into account the disturbances class limitation (2), one can write the inequalities for the disturbances $\xi(t)$ and its derivative

$$|\xi(t)| \leq \Sigma, |\dot{\xi}(t)| \leq \overline{\Sigma}, \tag{12}$$

where $\Sigma = \alpha_1\Gamma_b^0/J + \Gamma_b^1/J$, $\overline{\Sigma} = \alpha_1\Gamma_b^1/J + \Gamma_b^2/J$.

Let us formulate the main result of the paper.

Theorem 1. Let the parameters of inner controller (8)–(10) are chosen according to the inequalities

$$\begin{aligned}M_1 > \Sigma, \quad \alpha_1(M_1 - \Sigma) > 2\overline{\Sigma}, \ \beta_1 > 0, \ \beta_2 > 0;\\ \alpha_2 > 0, \ \alpha_2 > 0, \ M_2 > 0.\end{aligned} \tag{13}$$

Then the variables of the closed loop system (11) tends to zero exponentially.

Proof. Let us consider the proof of convergence the variable $\overline{\omega}$ only. The proof for variable $\overline{\psi}$ can be done in the same way. Firstly, a case when $\alpha_1^2/4 - \beta_1 \neq 0$ is considered. Let us introduce new coordinates

$$y_1 = \left|\alpha_1^2/4 - \beta_1\right|^{0.5}\overline{\omega}, \quad y_2 = \frac{\alpha}{2}\overline{\omega} + \widetilde{I}^*,$$

and rewrite the first equation of the system (11) in these variables

$$\begin{aligned}\dot{y}_1 &= -\frac{\alpha_1}{2}y_1 + \gamma_1 y_2,\\ \dot{y}_2 &= \gamma_2 y_1 - \frac{\alpha_1}{2}y_2 - M_1\,\mathrm{sign}(y_1) + \xi(t),\end{aligned}$$

where $\gamma_1 = \gamma_2 = \left|\alpha_1^2/4 - \beta_1\right|^{0.5}$ if $\alpha_1^2/4 - \beta_1 > 0$;

$\gamma_2 = -\gamma_1 = \left|\alpha_1^2/4 - \beta_1\right|^{0.5}$ if $\alpha_1^2/4 - \beta_1 < 0$.

The time derivative of the positive semidefinite Lyapunov function candidate

$$V_1 = |y_1| - \frac{\xi}{M_1}y_1 + \frac{y_1^2}{2M_1} + \frac{y_2^2}{2M_1} \tag{14}$$

along the trajectories of the system (11) is

$$\begin{aligned}\dot{V}_1 &= -\frac{\alpha_1}{2}|y_1| + \frac{\alpha_1\xi}{2M_1}y_1 - \frac{\dot{\xi}}{M_1}y_1 -\\ &-\frac{\alpha_1}{2M_1}y_1^2 + \frac{\gamma_1+\gamma_2}{M_1}y_1y_2 - \frac{\alpha_1}{2M_1}y_2^2 \leq\\ &\leq x^{\mathrm{T}}Qx - \overline{\alpha}|y_1| \leq \lambda_{\max}(Q)[y_1^2 + y_2^2] - \overline{\alpha}|y_1|\,,\end{aligned} \tag{15}$$

where $x^{\mathrm{T}} = (\,y_1 \ y_2\,)$, $Q = \begin{pmatrix} -\dfrac{\alpha_1}{2M_1} & \dfrac{\gamma_1+\gamma_2}{M_1}\\ \dfrac{\gamma_1+\gamma_2}{M_1} & -\dfrac{\alpha_1}{2M_1}\end{pmatrix}$,

$\overline{\alpha} = \dfrac{\alpha_1}{2}\left(1 - \dfrac{\Sigma}{M_1} - \dfrac{2\overline{\Sigma}}{\alpha_1 M_1}\right)$, $\lambda_{\max}(Q)$ is the maximal eigen value of the matrix Q.

It is easy to verify with the help of Sylvester's criterion that the quadratic form $x^{\mathrm{T}}Qx$ is negative definite $\forall\alpha_1 > 0$, $\beta_1 > 0$, $M_1 > 0$. From the condition of the theorem 1, $\overline{\alpha} > 0$ and V_1 is negative everywhere except the origin.

From (14) one can write

$$\begin{aligned}V_1 &\leq |y_1|\left(1 + \frac{\Sigma}{M_1}\right) + \frac{1}{2M_1}(y_1^2 + y_2^2) \leq\\ &\leq c_0(|y_1| + y_1^2 + y_2^2)\end{aligned}$$

where $c_0 = \max\left\{1 + \dfrac{\Sigma}{M_1}, \dfrac{1}{2M_1}\right\}$.

Taking into account the last inequality, equation (15) can be rewritten as

$$\begin{aligned}\dot{V}_1 &\leq \lambda_{\max}(Q)[y_1^2 + y_2^2] - \overline{\alpha}|y_1| \leq\\ &\leq -\overline{c}_1(|y_1| + y_1^2 + y_2^2) \leq -\gamma V_1,\end{aligned}$$

where $\gamma = \dfrac{\overline{c}_1}{c_0}$, $\overline{c}_1 = \min\{\,|\lambda_{\max}(Q)|,\ \overline{\alpha}\}$.

Further, the following estimation can be written for the variable y_1

$$|y_1(t)| \le V_0 e^{-\gamma(t-t_0)} \Rightarrow |\overline{\omega}(t)| \le Y_0 e^{-\gamma(t-t_0)},\ \forall t > t_0,$$

where t_0 is initial moment of time,

$$V_0 = |y_1(t_0)|\left(1+\frac{\Sigma}{M_1}\right)+\frac{1}{2M_1}([y_1(t_0)]^2+[y_2(t_0)]^2),$$
$$Y_0 = \left|\alpha_1^2/4-\beta_1\right|^{-0.5} V_0.$$

Now, the case $\alpha_1^2/4-\beta_1 = 0$ is considered. Using the coordinate transformation

$$\widetilde{y}_1 = \overline{\omega}, \widetilde{y}_2 = \frac{\alpha}{2}\overline{\omega}+\widetilde{I}^*,$$

the first two equation of the system (11) can be rewritten as

$$\begin{aligned}\dot{\widetilde{y}}_1 &= -\frac{\alpha_1}{2}\widetilde{y}_1+\widetilde{y}_2,\\ \dot{\widetilde{y}}_2 &= -\frac{\alpha_1}{2}\widetilde{y}_2 M_1\,\mathrm{sign}(\widetilde{y}_1)+\xi(t).\end{aligned} \tag{16}$$

The time derivative of the semidefinite Lyapunov function candidate

$$V_2 = \left(1+\frac{2}{\alpha_1^2}\right)\left[|\widetilde{y}_1|-\frac{\xi}{M_1}\widetilde{y}_1\right]+\widetilde{x}^{\mathrm{T}}\widetilde{Q}\widetilde{x}, \tag{17}$$

along the trajectories of the system (16) is

$$\begin{aligned}\dot{V}_2 &= \left(1+\frac{2}{\alpha_1^2}\right)\left(-\frac{\alpha_1}{2}|\widetilde{y}_1|-\frac{\dot{\xi}}{M_1}\widetilde{y}_1-\frac{\alpha_1}{2}\xi\,\widetilde{y}_1\right)-\\ &-\frac{1}{\alpha_1}|\widetilde{y}_1|+\frac{1}{\alpha_1 M_1}\xi\,\widetilde{y}_1-\frac{\alpha_1}{2M_1}(\widetilde{y}_1^2+\widetilde{y}_2^2)\le\\ &\le -\widetilde{\alpha}|\widetilde{y}_1|-\frac{\alpha_1}{2M_1}(\widetilde{y}_1^2+\widetilde{y}_2^2),\end{aligned} \tag{18}$$

where $\widetilde{Q} = \begin{pmatrix}\frac{1}{2} & \frac{1}{2\alpha_1}\\ \frac{1}{2\alpha_1} & \frac{1}{2}+\frac{1}{\alpha_1^2}\end{pmatrix}$ is positive definite matrix, since its eigen values $\lambda_1(\widetilde{Q}) = \dfrac{\sqrt{1+\frac{1}{\alpha_1^2}}\left(\sqrt{1+\frac{1}{\alpha_1^2}}-\frac{1}{\alpha_1}\right)}{2}$ and

$\lambda_2(\widetilde{Q}) = \dfrac{\sqrt{1+\frac{1}{\alpha_1^2}}\left(\sqrt{1+\frac{1}{\alpha_1^2}}+\frac{1}{\alpha_1}\right)}{2}$ are positive,

$$\widetilde{x}^{\mathrm{T}} = (\,\widetilde{y}_1\ \widetilde{y}_2\,),\ \ \widetilde{\alpha} = \left(1+\frac{2}{\alpha_1^2}\right)\times$$
$$\times\left[\frac{\alpha_1}{2}+\frac{\alpha_1}{\alpha_1^2+2}-\frac{\overline{\Sigma}}{M_1}-\left(\frac{1}{2}+\frac{1}{M_1(\alpha_1^2+2)}\right)\alpha_1\Sigma\right].$$

One can easy verify, that $\widetilde{\alpha} > 0$ under the condition of the theorem 1.

From (17) the following inequality can be written

$$V_2 \le \left(1+\frac{2}{\alpha_1^2}\right)\left(1+\frac{\Sigma}{M_1}\right)|\widetilde{y}_1|+\frac{\lambda_2(\widetilde{Q})}{M_1}(\widetilde{y}_1^2+\widetilde{y}_2^2)\le$$
$$\le \widetilde{c}_0(|\widetilde{y}_1|+\widetilde{y}_1^2+\widetilde{y}_2^2),$$

where $\widetilde{c}_0 = \max\left\{\left(1+\dfrac{2}{\alpha_1^2}\right)\left(1+\dfrac{\Sigma}{M_1}\right),\ \dfrac{\lambda_2(\widetilde{Q})}{M_1}\right\}.$

Combining (18) and the last inequality the upper derivative estimation can be derived

$$\dot{V}_2 \le -\widetilde{c}_1\left(|\widetilde{y}_1|+\widetilde{y}_1^2+\widetilde{y}_2^2\right)\le -\widetilde{\gamma}\,V_2,$$

where $\widetilde{c}_1 = \min\left\{\dfrac{\alpha_1}{2M_1},\ \widetilde{\alpha}\right\}$, $\widetilde{\gamma} = \dfrac{\widetilde{c}_1}{\widetilde{c}_0}$.

Finally, the following estimation can be written for the varia-bles $\widetilde{y}_1(t)$

$$\widetilde{y}_1 = \overline{\omega} = \widetilde{Y}_0 e^{-\widetilde{\gamma}(t-t_0)},\ \forall t > t_0,$$

where t_0 is initial moment of time,

$$\widetilde{Y}_0 = \left(1+\frac{2}{\alpha_1^2}\right)\left(1+\frac{\Sigma}{M_1}\right)|\widetilde{y}_1(t_0)|+\frac{\lambda_2(\widetilde{Q})}{M_1}[\widetilde{y}_1(t_0)]^2+$$
$$+\frac{\lambda_2(\widetilde{Q})}{M_1}[\widetilde{y}_2(t_0)]^2.$$

The proof of convergence for the variable $\overline{\psi}$ is the same.

Step 2. On the previous step we proof, that if parameters of the inner controller (8)–(10) are chosen according to inequalities (13) , than the output variables tend to zero exponentially. On the second step, we can choose real control inputs $u_{s\alpha},\ u_{s\beta}$ to provide synthesized relations. Equations (8) together constitute two simultaneous equations that may be solved for both the components of the stator current I, the results being as follows

$$\begin{aligned}I_d = \begin{pmatrix}I_{d\alpha}\\ I_{d\beta}\end{pmatrix} = \frac{1}{||\psi||}\begin{pmatrix}-\psi_{r\beta} & \psi_{r\alpha}\\ \psi_{r\alpha} & \psi_{r\beta}\end{pmatrix}\times\\ \times\begin{bmatrix}\dfrac{J}{c_5}[\widetilde{I}+\omega_d^{(1)}(t)]\\ \dfrac{c_3}{c_4}||\psi||+\dfrac{1}{2c_4}[\widetilde{\psi}+\psi_d^{(1)}(t)]\end{bmatrix},\end{aligned} \tag{19}$$

where the suffix, *d*, denotes a desired values of stator currents in α_β coordinate system.

To realize such desired stator current components the different switching technique can be applied based on modern PWM converters. From one hand, in α_β coordinate frame stator voltage components $u_\alpha,\ u_\beta$ may be chosen as discontinuous control inputs, and then they can be realized in the three-phase coordinate system with appropriate choice of time moments of switching of each voltage. From the other hand, the desired current components (19) may be recalculated in three-phase coordinate system and then used as reference values for PWM realization. Let us consider these two cases.

Case A. Discontinuous control synthesis in α_β coordinate system.

Now we are ready to establish the real control input. It is well known, that in induction motor the discontinuous control is used specifically to operate with an inverter. In this case using sliding mode theory is suitable Utkin et al. (2009). Due to this reason, let us introduce sliding variables

$$s_\alpha = I_\alpha - I_{d\alpha},\ s_\beta = I_\beta - I_{d\beta}. \tag{20}$$

The components of stator voltage are chosen in the discontinuous form

$$u_\alpha = -U_{0\alpha}\mathrm{sign}(s_\alpha),\ u_\beta = -U_{0\beta}\mathrm{sign}(s_\beta), \tag{21}$$

where $U_{0\alpha},\ U_{0\beta}$ are the constant power supply voltage.

The dynamics of sliding mode variables are governed by equations

$$\dot{s} = c_1[c_2 P(\omega)\psi - a_1(s+I_d) - U_0\mathrm{sign}(s)] - \dot{I}_d,$$

where $s = (s_\alpha,\ s_\beta)^{\mathrm{T}}$, $\mathrm{sign}(s) = [\mathrm{sign}(s_\alpha),\ \mathrm{sign}(s_\beta)]^{\mathrm{T}}$, $U_0 = \mathrm{diag}(U_{0\alpha},\ U_{0\beta})$.

Taking into account (19) and combining the last equation with the dynamics of the inner loop controller, one can write the full dynamic of the closed loop system

$$\begin{aligned} \dot{s} &= c_1[c_2P(\omega)\psi - a_1(s + I_d) - U_0\mathrm{sign}(s)] - \dot{I}_d, \\ \dot{\overline{\omega}} &= \widetilde{I} + \frac{c_5}{J}c_5\psi^{\mathrm{T}}T^{\mathrm{T}}s - \frac{\Gamma_L(t)}{J}, \\ \dot{\widetilde{I}} &= -\alpha_1\widetilde{I} - \beta_1\overline{\omega} - M_1\mathrm{sign}(\overline{\omega}), \\ \dot{\overline{\psi}} &= \widetilde{\psi} + 2c_4\psi^{\mathrm{T}}s, \\ \dot{\widetilde{\psi}} &= -\alpha_2\widetilde{\psi} - \beta_2\overline{\psi} - M_2\mathrm{sign}(\overline{\psi}). \end{aligned} \tag{22}$$

According to (10) if the conditions

$$U_{0\alpha} > \left| c_1[c_2(c_3\psi_\alpha + p\omega\psi_\beta) - a_1(s_\alpha + I_{d\alpha})] - \dot{I}_{d\alpha} \right|,$$

$$U_{0\beta} > \left| c_1[c_2(-p\omega\psi_\alpha + c_3\psi_\beta) - a_1(s_\beta + I_{d\beta})] - \dot{I}_{d\beta} \right|$$

are fulfilled, then sliding mode arises in the system (22) from the initial moment of time t_0 ($s(t) = 0$) and its dynamical order is reduced to the system (11), which is exponentially stable under conditions of the theorem 1.

Case B. Discontinuous control synthesis in three-phase coordinate system.

The desired stator currents in three phase coordinates can be calculated from (19) with the help of transformation matrix

$$I_{sd} = \begin{pmatrix} I_{sda} \\ I_{sdb} \\ I_{sdc} \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ -\frac{1}{2} & \frac{\sqrt{3}}{2} \\ -\frac{1}{2} & -\frac{\sqrt{3}}{2} \end{pmatrix} \begin{pmatrix} I_{d\alpha} \\ I_{d\beta} \end{pmatrix}, \tag{23}$$

where I_{sda}, I_{sdb}, I_{sdc} are the desired stator components in the three phase coordinate system.

From (23) one can choose the sliding surfaces in the form

$$s_a = I_{sa} - I_{sda},\ s_b = I_{sb} - I_{sdb},\ s_c = I_{sc} - I_{sdc}, \tag{24}$$

where $I_{sa},\ I_{sb},\ I_{sc}$ are measured stator components.

To organize sliding motion on the surfaces (24) the axes components of stator voltage $u_a,\ u_b,\ u_c$ are chosen in the form

$$\begin{gathered} u_a = -U_s\mathrm{sign}(s_a),\ u_b = -U_s\mathrm{sign}(s_b), \\ u_c = -U_s\mathrm{sign}(s_c), \end{gathered} \tag{25}$$

where U_s is the constant power supply voltage.
As in the previous case, if the value of U_s is sufficiently large, then sliding mode arises in the state of space of the system and the stated problem (6) is solved.

3.2 Automatic start algorithm

There are two singularities in the desired current components (19). First one is concerned with reference functions $\omega_d(t),\ \psi_d(t)$, and due to this reason the class of these functions is bounded by condition (5). Also, the system (3) can starts from zero initial conditions with $||\psi(0)|| = 0$. In this case from relations (19), (23) we get infinite desired values of stator current components. To avoid this problem we need to start our control algorithm from some moment of time $t_{\min}$, when

$$||\psi(t_{\min})|| > ||\psi||_{\min},$$

where $||\psi||_{\min}$ is some value of the rotor magnetic flux after which the automatic start algorithm is finished. For this reason designers must establish some automatic start algorithm, which provides non-zero rotor magnetic flux. For example, for the case (21) the maximum voltage is applied to both phase until the estimated rotor magnetic flux has grown sufficiently.
Thus, if

$$||\psi|| < ||\psi||_{\min},$$

then

$$u_\alpha = U_{0\alpha}\mathrm{sign}(I_\alpha),\ u_\beta = U_{0\beta}\mathrm{sign}(I_\beta). \tag{26}$$

According to this relation the angle of rotor flux has maximum growth speed to bisector of straight angle.

In the case (25), if

$$||\psi|| < ||\psi||_{\min},$$

then

$$u_{sa} = U_s\mathrm{sign}(I_a),\ u_{sb} = -U_s\mathrm{sign}(I_b),\ u_{sb} = -U_s\mathrm{sign}(I_b).$$

From (23) one can conclude, that in this case α axis component of rotor magnetic flux have the maximum growth speed.

4. SIMULATION

Let us consider the numerical simulation of the proposed controller (9), (19)–(21) for a three phase single pole pair induction motor, whose parameters are: $J = 0.375$ Kgm^2, $R_s = 5.3$ Ohm, $R_r = 3.3$ Ohm, $L_s = 0.365$ H, $L_r = 0.375$ H, $L_m = 0.34$ H. The reference signals for the rotor speed, magnetic flux and load torque are: $\omega_d(t) = 8\sin(4t) + 8\sin(8t)$ rad/s, $\psi_d(t) = 1 + 0.1\sin(4t)$ Wb, $\Gamma_L(t) = 6\sin(3t) + 2\sin(7t)$ Nm. The following values can be calculated for constants in (2): $\Gamma_b^0 = 6.5$ Nm, $\Gamma_b^1 = 26.5$ Nm/s, $\Gamma_b^2 = 152$ Nm/s^2. According to these bounds let us calculated values from (12): $\Sigma = 866.67\alpha_1 + 3534$, $\overline{\Sigma} = 3534\alpha_1 + 20267$. From conditions of the theorem 1 one can choose the following parameters for inner controller (9): $\alpha_1 = 10,\ \beta_1 = 20,$ $M_1 = 19800;\ \ \alpha_2 = 30,\ \beta_2 = 60,\ M_2 = 4400$. The voltages amplitudes for real control (21) are: $U_{0\alpha} = U_{0\beta} = 220$ V. The Dormand-Prince method is used for numerical experiments with fundamental sample time $t_s = 10^{-6}$ s. The automatic start algorithm is chosen in the form (26) with time delay is equal to t_s.

Fig. 1 shows the time histories of the rotor speed and flux modulus and the corresponding tracking errors.

The (α, β) components of stator current are reported in Fig. 2.

The convergence of electric motor torque to unknown external load torque and desired derivative of rotor reference speed according to equations of the closed loop system is shown in Fig. 3.

The next numerical experiments are provided for three fundamental sample times: $t_s = 10^{-5}$ s, $t_s = 10^{-6}$ s and $t_s = 10^{-7}$ s. The steady state errors for the rotor speed tracking and flux modulus are shown in Fig. 4 and Fig. 5. It is seen, that numerical values of the errors is proportional to fundamental sample time, which determines the switching frequency of the real control inputs (21). It is quite clear that the theoretical result of Theorem 1 is valid

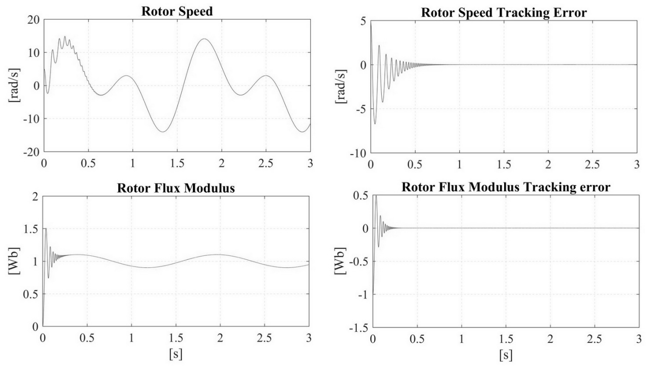

Fig. 1. Rotor speed, flux modulus and corresponding tracking errors.

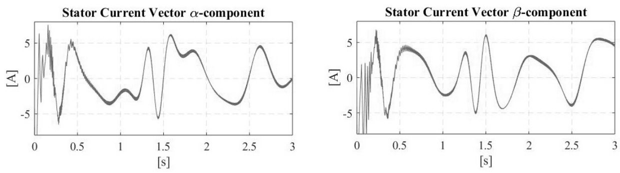

Fig. 2. (α, β) components of stator current vectors.

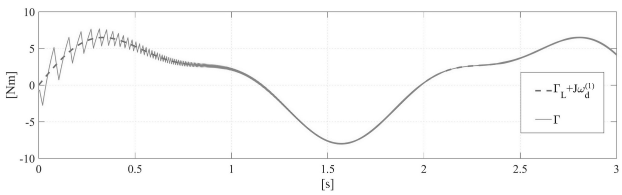

Fig. 3. Electric motor torque and load torque.

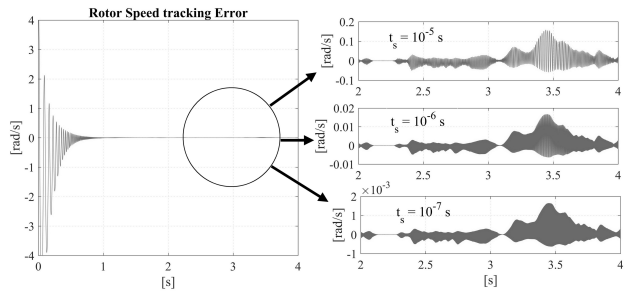

Fig. 4. Rotor speed tracking error for different sample times.

only for the infinite frequency of relay switching ($t_s = 0$). In real situation we have some output errors determined by relays switching frequency as in classical sliding mode regime Utkin et al. (2009).

5. CONCLUSION

The independent control of rotor magnetic flux and rotor speed control was considered in the paper under influence of unknown unmatched external load torque. The proposed control algorithm is based on discontinuous relay function, which can be realized easily with the help of modern power converters. The proposed control law can be used in many practical applications. The robust problem statement with unknown parameters deviations such as rotor resistance, rotor inertia changing must be considered in the future.

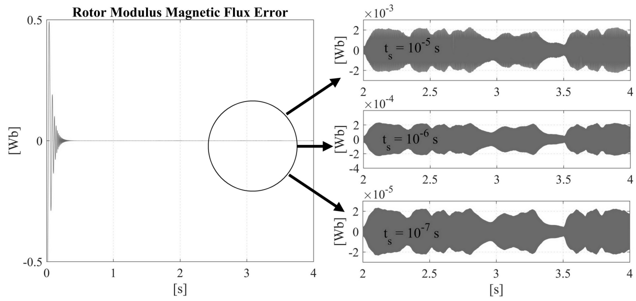

Fig. 5. Flux modulus tracking error for different sample times.

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